3. Letter Strings

In abstract algebra, letters do not stand for numbers. Abstract algebra has many applications, for example to particle physics, or to the analysis of the Rubik's cube. Here is a simple example.

The YZ Game

In this game, the object is, starting with a string of Ys and Zs, to simplify the string by following strict rules. The rules are:

YYY can be erased
ZZ can be erased
the commutative law: YZ = ZY

Examples:

a.
```
YZZYYZYZYYZ   (erase ZZ)
Y   YYZYZYYZ  (erase YYY)
      ZYZYYZ  (commute YZ)
      ZZYYYZ  (erase ZZ and YYY)
            Z (can't be simplified)
```

b.
```
ZYYYZ  (erase YYY)
Z   Z  (erase ZZ)
  E    (the empty string is left)
```

1. Simplify the strings:
   a. YZYZZYYZ b. YYYYZZYZY
   c. YZYZYZYZYZYZYZYZZZYZYZYYZY

Including the empty string E, there are 6 essentially different strings that cannot be simplified. They are called the *elements of the YZ group*.

2. Find all the elements of the YZ group.

The symbol ↔ represents the operation "put together and simplify". For example:

Y ↔ YY = E
YZ ↔ YZ = YY
Y ↔ E = Y

3. Compute:
   a. E ↔ YZ b. YZ ↔ YY c. Z ↔ YZ

4. Find the missing term:
   a. YZ ↔ ___ = E b. Z ↔ ___ = YZ c. YY ↔ ___ = Z

For the YZ group, ↔ works a little bit like multiplication. Another way to write the first two rules is:
$Y^3 = E$ and $Z^2 = E$

5. The only powers of `Y` are: `Y`, Y^2, and `E`. Explain.

6. Find *all* the powers of each element of the YZ group.

7. Simplify (show your work):
   a. Y^{1000} b. $(YZ)^{1001}$

8. Make a "↔ table".

↔	E	Y	YY	Z	YZ	YYZ
E						
Y						
YY						
Z						
YZ						
YYZ						

9. What element of the group works like 1 for multiplication?

10. What is the reciprocal of each element? (In other words, for each element, what element can be put together with it to get the "1"?)

The yz Game

For this group, the rules are:
`yyy` can be erased
`zz` can be erased
`yzy = z`
The empty string is called `e`.
There is no commutative law.

11. * Do Problems 1-10 for the yz group. (Hint: `zyy` and `yyz` *can* be simplified.)

12. **Report:** Write a report on the yz group.